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Evaluation of Methods for Estimating the Decay Constant (*K*) of Horton's Infiltration Model

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Abstract: **Horton's equation is one of the earliest and most popular empirical models for simulating infiltration. The aim of this paper is to evaluate the method for determining Horton's decay constant from three (3) methods to determine their simplicity and accuracy. Six (6) infiltration tests was carried out and the average of the six tests was used for determining the steady state infiltration rate i_c = 4cm/hr and initial infiltration rate i_o = 34cm/hr. Three methods were used to evaluate the decay constant k, Method I and III gave same result 0.8099 and the result of Method II is 0.8086. It is clear that there is no significant difference between the results, method II proved to be error prone. Method I is highly recommended above Method II and III because it is less disposed to error and simple to calculate.**

Keywords: **Decay constant, linear regression, infiltration capacity, Horton's model.**

1. INTRODUCTION

Infiltration is the process by which water seeps into the ground through the surface of the earth (Beven, 2004; Thornes, 2009). Horton's equation is the most popular empirical model for the simulation of infiltration. The equation named after Robert E. Horton (1940), is a semi-empirical formula that says that infiltration starts at a constant rate i_0, and is decreasing exponentially with time *t*. After some time when the soil saturation level reaches a certain value, the rate of infiltration will level off to the rate i_c. The infiltration rate is given by:

$$i = i_c + (i_0 - i_c)e^{-kt} \text{ ------------Eq. [1]}$$

Then the cumulative Infiltration is the integral of Equation [1]

$$I = i_c t + \frac{i_0 - i_c}{k}[1 - e^{-kt}] \text{ ----------------Eq. [2]}$$

Where *I* is the cumulative infiltration, *i* is the infiltration rate at time *t*; i_o is the initial infiltration rate, i_c is the constant or equilibrium infiltration rate after the soil has been saturated or minimum infiltration rate; *k* is the decay constant specific to the soil. Different methods have been used by different researchers for determining the equation's parameters, this paper is a review of the methods for determining Horton's decay constant (*k*) with a specific aim to assess the most suitable, simple and accurate method.

2. MATERIALS AND METHODS

Data Collection and Analysis:

Secondary cumulative infiltration data was taken from Ajayi (2015) as shown in Table 1 below, and the average was used to estimate the parameters of the models.

Table1: Average values from the 6 infiltration tests

Time (hr)	Cumulative Infiltration *I* (cm)	Infiltration rate *i* (cm/hr)
0.05	1.57	31.33
0.08	2.40	28.80
0.17	3.97	23.80
0.33	6.00	18.00
0.50	7.27	14.53
0.75	9.10	12.13
1.00	10.73	10.73
1.50	12.93	8.62
2.00	14.50	7.25
2.50	16.17	6.47
3.00	17.57	5.86
3.50	18.77	5.36
4.00	19.37	4.84

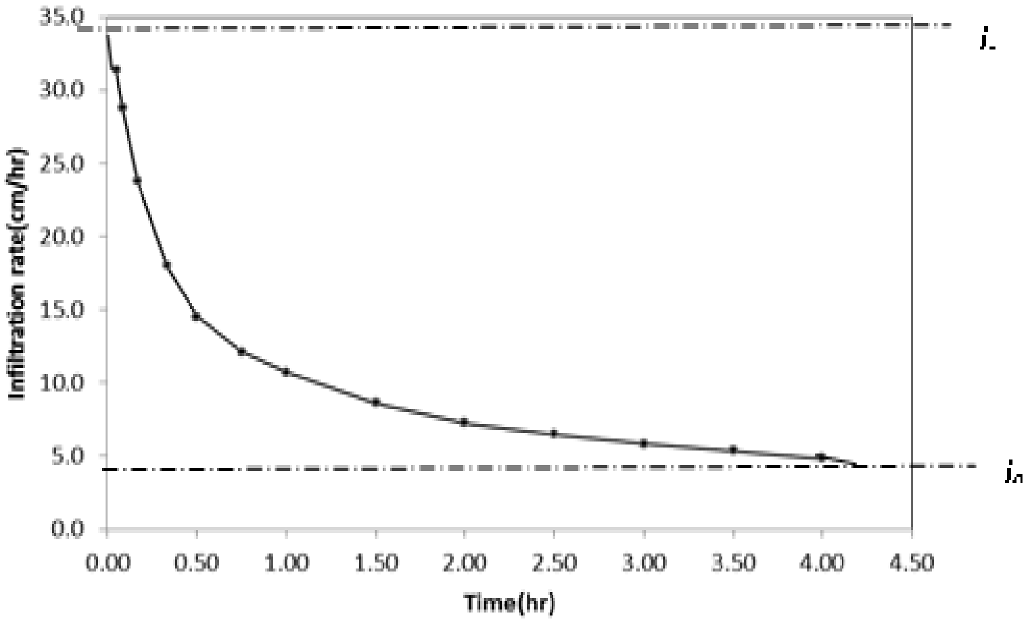

Fig. 1: Infiltration rate and the elapsed time

From the graph, i_0 = 4.0cm/hr and i_c = 34cm/hr

Methods of evaluating infiltration parameters:

Three (3) methods for determining the equation parameters will be studied in this paper.

Method I:

The most common method employed by Mudiare *et al.*, (2011). In this method, simple regression procedure was adapted to estimate the parameters of the Horton infiltration model from the field measured values, which involves the determination of *k* value as the slope of the line of best fit between *lny* and elapsed time (t).

From Eq. 1, Taking i_c to the left hand side gives:

$i - i_c = (i_0 - i_c)e^{-kt}$ -------Eq. [3]

Dividing through by $i_0 - i_c$, gives:

$\left(\frac{i - i_c}{i_0 - i_c}\right) = e^{-kt}$-------Eq. [4]

$\left(\frac{i - i_c}{i_0 - i_c}\right) = e^{-kt}$---------Eq. [5]

Taking the Logarithm of both sides gives:

$ln\left(\frac{i - i_c}{i_0 - i_c}\right) = -kt$---------Eq. [6]

Let $y = \left(\frac{i - i_c}{i_0 - i_c}\right)$

$lny = -kt$---------Eq. [7]

To solve for the equation, *i* was taken as the measured infiltration rate at any given time *t*. Computing the values of the initial (i_o) and final steady-state infiltration (i_c) in the left hand-side. The plot of lny versus the elapsed time (t) was obtained on a linear graph of which the slope gives the value of k which is the third parameter of the equation.\

Table 2: Derivation of *ln*y

Time(hr)	*i*(cm/hr)	$i - i_c$	$i_o - i_c$	$y = i - i_c / i_o - i_c$	ln (y)
0.05	31.33	27.33	30	*0.911*	-0.093
0.08	28.80	24.80	30	0.827	-0.190
0.17	23.80	19.80	30	0.660	-0.416
0.33	18.00	14.00	30	0.467	-0.762
0.50	14.53	10.53	30	0.351	-1.047
0.75	12.13	8.13	30	0.271	-1.305
1.00	10.73	6.73	30	0.224	-1.494
1.50	8.62	4.62	30	0.154	-1.870
2.00	7.25	3.25	30	0.108	-2.223
2.50	6.47	2.47	30	0.082	-2.498
3.00	5.86	1.86	30	0.062	-2.783
3.50	5.36	1.36	30	0.045	-3.092
4.00	4.84	0.84	30	0.028	-3.574

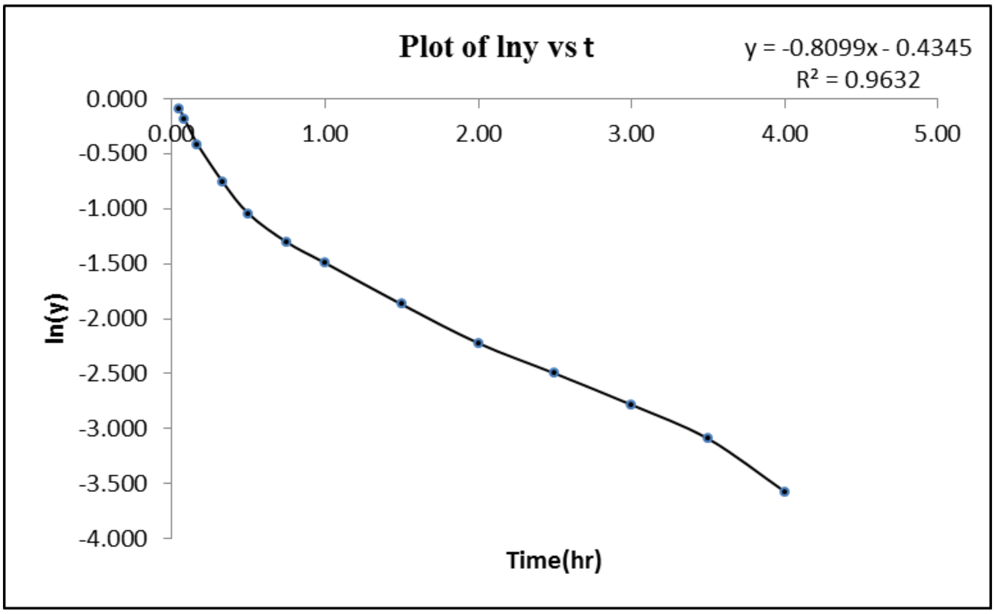

Fig2: The graph of *ln*y and the elapsed time

From the graph above of the relation $lny = -kt$, the slope is the value of *k* which is equal to 0.4355. Substituting, i_0 = 4, i_c = 34, k = -0.8099 and the corresponding values of t into the equation, we'll have:

$i = 4 + (34 - 4) * e^{-0.8099t}$------- Eq. [8]

And the equation:

$I = 4t + \frac{34-4}{0.8099}[1 - e^{-0.8099t}]$ ----------------Eq. [9]

Method II:

The second method studied was employed by Reddy (2004) and Raghunath (2006), it involves analytical determination of the area under the infiltration curve. From Eq. 1

Taking i_c to the left hand side we have:

$i - i_c = (i_o - i_c)e^{-kt}$

Integrating the right hand and the left hand side separately, gives:

LHS ----->> $\int_0^\infty (i - i_c).dt = I_c$

Where I_c is the area of the shaded portion of the plot between *i* (cm/hr) and t (hr)

RHS ----->> $\int_0^\infty (i_0 - i_c)e^{-kt}.dt = \frac{i_0 - i_c}{k}$

Equating both equations, gives:

$F_c = \frac{i_0 - i_c}{k}$----------Eq. [10]

Making *k*, the subject of the formula gives:

$k = \frac{i_0 - i_c}{I_c}$----------Eq. [11]

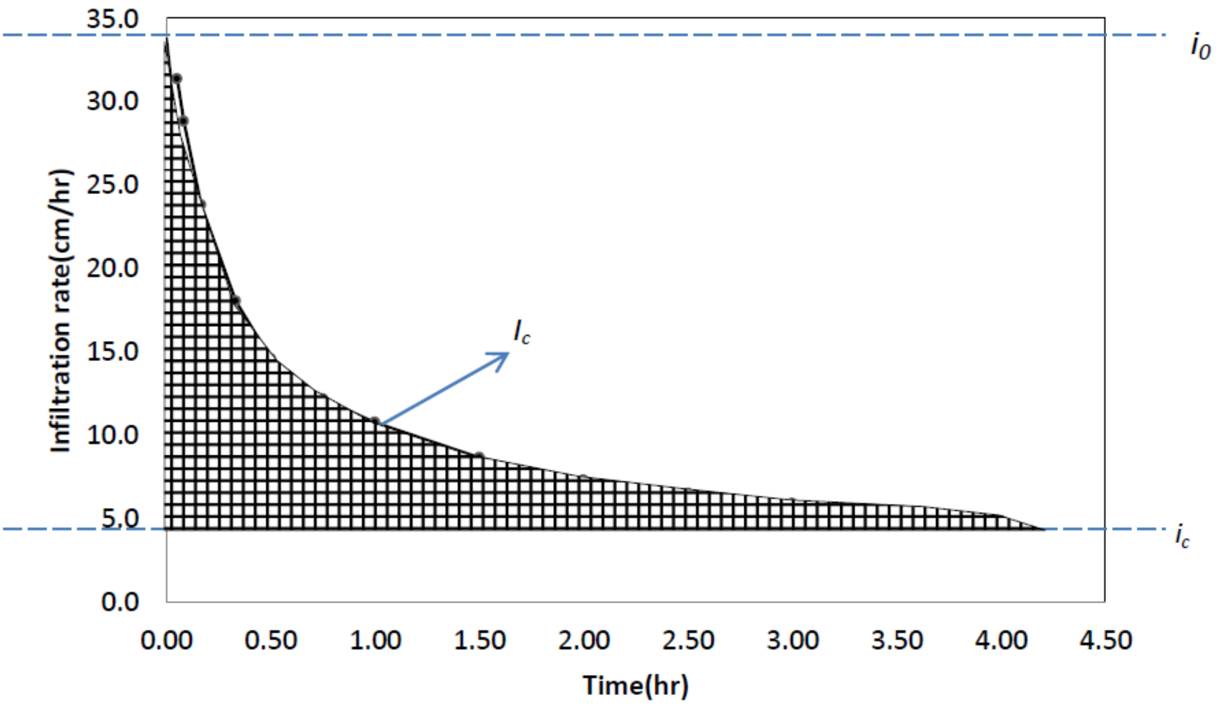

Figure 3: The graph of infiltration rate and the elapsed time to get I_c

From the graph above, on the x axis, 0.5 hr = 1 unit and on the y axis 5 cm/hr = 1 unit, the area of the big box will be (0.5 hr*5 cm/hr) = 2.5 cm square units or the small box will be 0.1 hr*1 cm/hr which gives 0.1 cm square units. The number of small boxes multiply by 0.1cm square gives us the area of the shaded portion I_c. From the graph the area of the shaded portion from the number of small boxes are approximately 371. Multiplying by 0.1 cm gives 37.1 cm.

Therefore from Eq. [11]

$k = \frac{34-4}{37.1} = \frac{30}{37.1} = 0.8086 \approx 0.8090$

Method III:

The third method studied was also employed by Reddy (2004).

$$i = i_c + (i_0 - i_c)e^{-kt}$$

Taking i_c to the left hand side we have:

$i - i_c = (i_0 - i_c)e^{-kt}$

Taking the natural logarithm of both sides we have:

$\ln(i - i_c) = ln(i_0 - i_c) - kt$

Let y = $\ln(i - i_c)$ and c = $ln(i_0 - i_c)$, the equation becomes:

$y = -kt + c$ Or $y = c - kt$

Using analytical method to solve for the value of the slope; from Table 3 below:

$r = \left[\frac{(y - \hat{y}) * (t - \check{T})}{n}\right] / S_t * S_y$

$k = \frac{r * S_y}{S_t}$

Where: r = correlation coefficient, S_t = the standard deviation of elapsed time, S_y = the standard deviation y, ŷ = mean of y, Ť = mean of the time.

Table 3: Derivation of r and k

Time(hr)	***i(cm/hr)***	***i - ic***	***y=ln(i - ic)***	***(y-ŷ)²***	***(t- Ť)²***	***(y-Y)*(t-T)***
0.05	31.33	27.33	3.31	2.40	2.07	-2.23
0.08	28.80	24.80	3.21	2.10	1.98	-2.04
0.17	23.80	19.80	2.99	1.50	1.75	-1.62
0.33	18.00	14.00	2.64	0.77	1.34	-1.02
0.50	14.53	10.53	2.35	0.35	0.98	-0.59
0.75	12.13	8.13	2.10	0.11	0.55	-0.25
1.00	10.73	6.73	1.91	0.02	0.24	-0.07
1.50	8.62	4.62	1.53	0.05	0.00	0.00
2.00	7.25	3.25	1.18	0.34	0.26	-0.30
2.50	6.47	2.47	0.90	0.73	1.02	-0.87
3.00	5.86	1.86	0.62	1.30	2.28	-1.72
3.50	5.36	1.36	0.31	2.11	4.04	-2.92
4.00	4.84	0.84	-0.17	3.73	6.30	-4.85
Ť=1.49			Ŷ = 1.76			Mean = -1.42

The standard deviations are: S_t = 1.325 and S_y = 1.093

$r = \frac{-1.42}{S_t * S_y} = \frac{-1.42}{1.325 * 1.093} = -0.9814$

$k = \frac{-0.9814 * 1.093}{1.325}$

$k = \frac{-1.0717}{1.325}$

$k = -0.8099$

3. RESULT, DISCUSSION AND CONCLUSION

The table below shows the result from the use of the three methods

	Method I	Method II	Method III
k value	0.8099	0.8086	0.8099

From the result above, Method I and III gave same result, although, the values of the three methods vary, it is seen clearly that there is no significant difference between them, the challenge with method II is the fact that it is very error prone, because of its complexity in analysis, care has to be taken when finding the area under the curve, in this study, it was done by counting the boxes and that is where the deviation from 0.8099 arose. Conclusively, Method I is highly recommended above Method III because it is less error prone and simple to calculate.

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